Universidad Autónoma de Baja California

Facultad de Ingeniería, Arquitectura y Diseño

INGENIERÍA EN NANOTECNOLOGÍA

ETAPA DISCIPLINARIA

TAREAS-TALLER

33543 CAMPOS ELECTROMAGNÉTICOS

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UNIVERSIDAD AUTÓNOMA DE BAJA CALIFORNIA

FACULTAD DE INGENIERÍA, ARQUITECTURA Y DISEÑO
INGENIERÍA EN NANOTECNOLOGÍA

- CAMPOS ELECTROMAGNÉTICOS -

Problems 1: Electric Field
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1. Two point charges, $Q_1 = 50\mu$C and $Q_2 = 10\mu$C, are located at $(-1, 1, -3)$ m and $(3, 1, 0)$ m, respectively. Find the force at Q_1.

2. Find the force on a 100μC charge at $(0, 0, 3)$ m if four like charges of 20μC are located on the x and y axes at ± 4 m.

3. Point charge $Q_1 = 300\mu$C, located at $(1, -1, -3)$ m, experiences a force $\vec{F} = 8\,\hat{a}_x - 8\,\hat{a}_y + 4\,\hat{a}_z$ N, due to charge Q_2 at $(3, -3, -2)$ m. Determine Q_2.

4. Ten identical charges of 500μC each are spaced equally around a circle of radius 2 m. Find the force on a charge of -20μC located on the axis, 2 m from the plane of the circle.

5. Two uniform line charges of $\rho_\ell = 4$ nC/m each are parallel to the z axis at $x = 0$, $y = \pm 4$ m. Determine the electric field $\vec{E}$ at $(\pm 4, 0, z)$.

6. Determine $\vec{E}$ at the origin due to a uniform line charge distribution with $\rho_\ell = 3.30$ nC/m located at $x = 3$ m, $y = 4$ m.

7. The plane $-x + 3y - 6z = 6$ contains a uniform charge distribution $\rho_S = 0.53$ nC/m^2. Find $\vec{E}$ on the side containing the origin.

8. Two infinite sheets of uniform charge density $\rho_S = 10^{-9}/6\pi$ C/m^2 are located at $z = -5$ m and $y = -5$ m. Determine the uniform line charge density ρ_ℓ necessary to produce the same value of $\vec{E}$ at $(4, 2, 2)$ m, if the line charge is located at $z = 0$, $y = 0$.

9. A circular ring of charge with radius 2 m lies in the $z = 0$ plane, with center at the origin. If the uniform charge density is $\rho_\ell = 10$ nC/m, find the point charge Q at the origin which produce the same electric field $\vec{E}$ at $(0, 0, 5)$ m.

10. A circular disk $r \leq 2$ m in the $z = 0$ plane has a charge density $\rho_S = 10^{-8}/r$ C/m^2. Determine the electric field $\vec{E}$ for the point $(0, \phi, h)$.

11. A circular disk $r \leq 1$ m, $z = 0$ has a charge density $\rho_S = 2(r^2 + 25)^{3/2} e^{-10r}$ C/m^2. Find $\vec{E}$ at $(0, 0, 5)$ m.

12. Two uniform charge distributions are as follows: a sheet of uniform charge density $\rho_s = -50$ nC/m^2 at $y = 2$ m and a uniform line of $\rho_\ell = 0.2$ μC/m at $z = 2$ m, $y = -1$ m. At what points in the region will $\vec{E}$ be zero?

13. A finite sheet of charge, of density $\rho_s = 2x(x^2 + y^2 + 4)^{3/2}$ (C/m^2), lies in the $z = 0$ plane for $0 \le x \le 2$ m and $0 \le y \le 2$ m. Determine $\vec{E}$ at $(0, 0, 2)$ m.

14. Charge is distributed with constant density ρ_v throughout a spherical volume of radius a. Show that

$$\vec{E} = \left| \begin{array}{l} \dfrac{r\rho_v}{3\epsilon_0}\hat{a}_r \ ; \ r \le a \\ \\ \dfrac{a^3\rho_v}{3\epsilon_0 r^2}\hat{a}_r \ ; \ r \ge a \end{array} \right.$$

15. Point charges 1 mC and -2 mC are located at $(3, 2, -1)$ and $(-1, -1, 4)$ respectively. Calculate the electric force on a 10 nC charge located at $(0, 3, 1)$ and the electric field intensity at that point.

16. Point charges 5 nC and -2 nC are located $(2, 0, 4)$ and $(-3, 0, 5)$, respectively.
    (a) Determine the force on a 1 nC point charge located at $(1, -3, 7)$.
    (b) Find the electric field $\vec{E}$ at $(1, -3, 7)$.

17. A circular ring of radius a carries a uniform charge ρ_ℓ C/m and is placed on the xy-plane with axis the same as the z-axis.
    (a) Show that

$$\vec{E}(0, 0, h) = \frac{\rho_\ell a h}{2\epsilon_0[h^2 + a^2]^{3/2}}\,\hat{a}_z$$

    (b) What values of h gives the maximum value of $\vec{E}$?
    (c) If the total charge on the ring is Q, find $\vec{E}$ as $a \to 0$.

18. The finite sheet $0 \le x \le 1$, $0 \le y \le 1$ on the $z = 0$ plane has a charge density $\rho_s = xy(x^2 + y^2 + 25)^{3/2}$ nC/m^2. Find
    (a) The total charge on the sheet.
    (b) The electric field at $(0, 0, 5)$.
    (c) The force experimented by a -1 mC charge located at $(0, 0, 5)$.

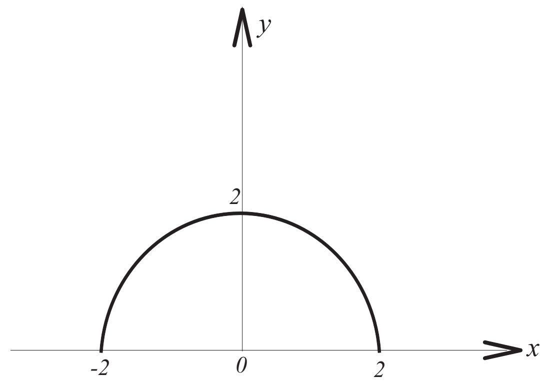
19. A square plate described by $-2 \le x \le 2$, $-2 \le y \le 2$, $z = 0$ carries a charge $12 \mid y \mid$ mC/m^2. Find the total charge on the plate and the electric field intensity at $(0, 0, 10)$.

20. Planes $x = 2$ and $y = 3$, respectively, carry charges 10 nC/m^2 and 15 nC/m^2. If the line $x = 0$, $z = 2$ carries charge 10π nC/m, calculate $\vec{E}$ at $(1, 1, -1)$ due to the three charge distributions.

21. Planes $x = 2$ and $y = 3$, respectively, carry charges 10 nC/m^2 and 15 nC/m^2. If the line $x = 0$, $z = 2$ carries charge 10π nC/m is rotated through 90^0 about the point $(0, 2, 2)$ so that it become $x = 0$, $y = 2$, find $\vec{E}$ at $(1, 1, -1)$.

22. Charges $+Q$ and $+3Q$ separated by a distance 2 m. A third charge is located such that the electrostatic system is in equilibrium. Find the location and the value of the third charge in terms of Q.

23. A point charge Q is located at point $P(0, -4, 0)$, while a 10 nC charge is uniformly distributed along a circular ring as shown in the next Figure. Find the value of Q such that $E(0, 0, 0) = 0$.



24. (a) Show that the electric field at point $(0, 0, h)$ due to the rectangle described by $-a \leq x \leq a$, $-b \leq y \leq b$, $z = 0$ carrying uniform charge ρ_s C/m^2 is

$$\vec{E}(0,0,h) = \frac{\rho_s}{\pi\epsilon_0} tan^{-1}[\frac{ab}{h(a^2+b^2+h^2)^{1/2}}]\ \hat{a}_z$$

(b) If $a = 2$, $b = 5$, $\rho_s = 10^{-5}$, find the total charge on the plate and the electric field intensity at $(0, 0, 10)$.

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FACULTAD DE INGENIERÍA, ARQUITECTURA Y DISEÑO
INGENIERÍA EN NANOTECNOLOGÍA
- CAMPOS ELECTROMAGNÉTICOS -

Problems 2: Gauss's law
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1. Charge is distributed in the spherical region $r \leq 2$ m with density $\rho = \frac{-200}{r^2}$ (μC/m^3). What net flux crosses the surfaces $r = 1$ m, $r = 4$ m, $r = 500$ m?

2. If a point charge Q is at the origin, find an expression for the flux which crosses the portion of a sphere, centered at the origin, described by $\alpha \leq \phi \leq \beta$.

3. A uniform line charge with $\rho_\ell = 3$ μC/m lies along the x axis. What flux crosses a spherical surface centered at the origin with $r = 3$ m?

4. Given that $\vec{D} = 500e^{-0.1x}\,\hat{a}_x$ (μC/m^2), find the flux Ψ crossing surfaces of area 1 m^2 normal to the x axis located at $x = 1$ m, $x = 5$ m, and $x = 10$ m.

5. Given a charge distribution with density $\rho = 5r$ (C/m^3) in spherical coordinates, use Gauss's law to find $\vec{D}$.

6. Given $\vec{D} = \frac{10}{r^2}\left[1 - e^{-2e}(1 + 2r + 2r^2)\right]\hat{a}_r$ in spherical coordinates, find the charge density.

7. In the region $a \leq r \leq b$ (cylindrical coordinates) $\vec{D} = \rho_0\left(\frac{r^2 - a^2}{2r}\right)\hat{a}_r$, and for $r > b$, $\vec{D} = \rho_0\left(\frac{b^2 - a^2}{2r}\right)\hat{a}_r$. For $r < a$, $\vec{D} = 0$. Find ρ in all three regions.

8. Given $\vec{D} = \left(\frac{5r^2}{4}\right)\hat{a}_r$ (C/m^2), in spherical coordinates, evaluate both sides of the divergence theorem for the volume enclosed by $r = 4$ m and $\theta = \pi/4$.

9. Given that $\vec{D} = 2r\cos(\phi)\,\hat{a}_\phi - \frac{\sin(\phi)}{3r}\,\hat{a}_z$ in cylindrical coordinates, find the flux crossing the portion of the $z = 0$ plane defined by $r \leq a$, $0 \leq \phi \leq \pi/2$. Repeat for $3\pi/2 \leq \phi \leq 2\pi$. Assume flux is positive in the $\hat{a}_z$ direction.

10. A point charge, $Q = 2000$ pC, is at the origin of spherical coordinates. A concentric spherical distribution of charge at $r = 1$ m has a charge density $\rho_s = 40\pi$ pC/m^2. What surface charge density on a concentric shell at $r = 2$ m would result in $\vec{D} = 0$ for $r > 2$ m?

11. An electrostatic field is given by $\vec{E} = \lambda(x\,\hat{a}_x + y\,\hat{a}_y)$ where λ is a constant. Use Gauss's law to find the total charge enclosed by the surface consisting of S_1, the curved portion of the half-cylinder $z = (r^2 - y^2)^{1/2}$ of length h; the two semi circular plane end pieces, S_2 and S_3; and S_4 the rectangular portion of the xy-plane. Express your result in terms of λ, r and h.

12. In the region $0 < r < 1$ m, $\vec{D} = (-2\text{x}10^{-4}/r)\ \hat{a}_r$ (C/m^2) and for $r > 1$ m, $\vec{D} = (-4\text{x}10^{-4}/r^2)\ \hat{a}_r$ (C/m^2) in spherical coordinates. Find the charge density in both region.

13. Charge in the form of a plane sheet with density $\rho_s = 40$ (μC/m^2) is located at $z = -0.5$ (m). A uniform line charge of $\rho_\ell = -6$ (μC/m) lies along the $y-$axis. What net flux crosses the surface of a cube 2 (m) on an edge, centered at the origin of coordinate system?

14. Determine the flux crossing a 1 mm by 1 mm area on surface of a cylindrical shell at $r = 10$ m, $z = 2$ m, $\psi = 53.3^o$ if $\vec{D} = 2x\ \hat{a}_x + 2(1 - y)\ \hat{a}_y + 4z\ \hat{a}_z$.

UNIVERSIDAD AUTÓNOMA DE BAJA CALIFORNIA
FACULTAD DE INGENIERÍA, ARQUITECTURA Y DISEÑO
INGENIERÍA EN NANOTECNOLOGÍA
- CAMPOS ELECTROMAGNÉTICOS -

Problems 3: Electric Potential
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1. An electrostatic field is given by $\vec{E} = (\frac{x}{2} + 2y)\,\hat{a}_x + 2x\,\hat{a}_y$. Find the work done in moving a point charge $Q = -20\mu$C
   (a) From the origin to $(4, 0, 0)$ m.
   (b) From $(4, 0, 0)$ m to $(4, 2, 0)$ m.
   (c) From $(4, 2, 0)$ m to $(0, 0, 0)$ m.

2. Two point charges -4μC and 5μC are located at $(2, -1, 3)$ and $(0, 4, -2)$, respectively. Find the potential at $(1, 0, 1)$.

3. A total charge of 40/3 nC is uniformly distributed in the form of a circular disk of radius 2m. Find the potential due to the charge at point of the axis, 2m from the disk.

4. A point charge of 5 nC is located at $(-3, 4, 0)$, while line $y = 1$, $z = 1$ carries uniform charge 2 nC/m.
   (a) If $V = 0$ V at $O(0, 0, 0)$, find V at $A(5, 0, 1)$.
   (b) If $V = 100$ V at $B(1, 2, 1)$, find V at $C(-2, 5, 3)$.
   (c) If $V = -5$ V at O, find V_{BC}.

5. Given the electric field $\vec{E} = 2x\,\hat{a}_x - 4y\,\hat{a}_y$ $(\frac{V}{m})$, find the work done in moving a point charge $+2$ C
   (a) from $(2, 0, 0)$ m to $(0, 0, 0)$ m and then from $(0, 0, 0)$ m to $(0, 2, 0)$ m;
   (b) from $(2, 0, 0)$ m to $(0, 2, 0)$ m along the straight-line path joining the two points.

6. Given the field $\vec{E} = (k/r)\,\hat{a}_r$ $(\frac{V}{m})$ in cylindrical coordinates, show that the work needed to move a point charge Q from any radial distance r to a point at twice that radial distance is independent of r.

7. For a line charge $\rho_\ell = (10^{-9}/2)$ C/m on the z axis, find V_{AB}, where A is $(2\,\text{m}, \pi/2, 0)$ and B is $(4\,\text{m}, \pi, 5\,\text{m})$.

8. Given the field $\vec{E} = (-16/r^2)\,\hat{a}_r$ $(\frac{V}{m})$ in spherical coordinates, find the potential of point $(2\,\text{m}, \pi, \pi/2)$ with respect to $(4\,\text{m}, 0, \pi)$

9. Find the work done in moving a point charge $Q = -20\,\mu$C from the origin to $(4, 2, 0)$ m in the field $\vec{E} = 2(x + 4y)\,\hat{a}_x + 8x\,\hat{a}_y$ $(\frac{V}{m})$ along the path $x^2 = 8y$.

10. Find the work done in moving a point charge $Q = 3\mu$C from $(4\text{m}, \pi, 0)$ to $(2\text{m}, \pi/2, 2\text{m})$, cylindrical coordinates, in the field $\vec{E} = (10^5/r)\,\hat{a}_r + (10^5)z\,\hat{a}_z$ $(\frac{V}{m})$.

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INGENIERÍA EN NANOTECNOLOGÍA
- CAMPOS ELECTROMAGNÉTICOS -

Problems 4: Magnetostatics
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1. Show that the magnetic field due to the finite current element shown in Fig. 1 is given by $\vec{H} = \frac{I}{4\pi r}(\sin\alpha_1 - \sin\alpha_2)\,\hat{a}_\phi$.

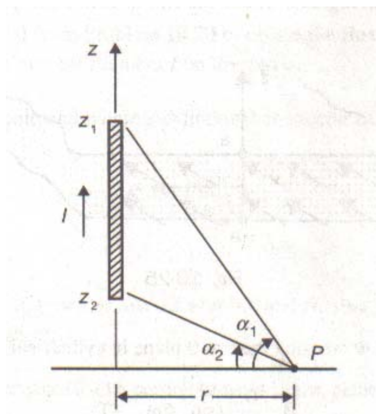

Figure 1: Finite current element.

2. Obtain $d\vec{H}$ at a general point (r, θ, ϕ) in spherical coordinates, due to a differential current element $Id\vec{\ell}$ at the origin in the positive z direction.

3. In cylindrical coordinates, $\vec{J} = 10^5(\cos^2 2r)\,\hat{a}_z$ in a certain region. Obtain $\vec{H}$ from this current density and then take the curl of $\vec{H}$ and compare with $\vec{J}$.

4. One uniform sheet, $\vec{K} = K_0\,\hat{a}_y$, is at $z = b > 2$ and another, $\vec{K} = K_0\,(-\hat{a}_y)$, is at $z = -b$. Find the magnetic flux crossing the area defined by $-2 \leq x \leq 2$, $0 \leq y \leq L$. Assume free space.

5. Currents in the inner and outer conductors of Fig. 2 are uniformly distributed. Use Ampère's law to show that for $b \leq r \leq c$, $\vec{H} = \frac{I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right)\hat{a}_\phi$.

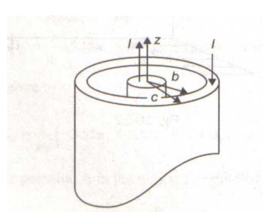

Figure 2: Currents in the inner and outer conductors.

6. A current filament of 10 A in the $+y$ direction lies along the y axis, and a current sheet, $\vec{K} = 2.0\,\hat{a}_x$ A/m, is located at $z = 4$ m. Determine $\vec{H}$ at the point $(2, 2, 2)$.

7. A cylindrical conductor of radius 10^{-2} m has an internal magnetic field $\vec{H} = (4.77 \times 10^4)(\frac{r}{2} - \frac{r^2}{3\times 10^{-2}})\,\hat{a}_\phi$ (A/m). What is the total current in the conductor?

8. In Cartesian coordinates, a constant current density, $\vec{J} = J_0\,\hat{a}_y$, exist in the region $-a \leq z \leq a$. See Fig. 3. Use Ampère's law to find $\vec{H}$ in all regions. Obtain the curl of $\vec{H}$ and compare with $\vec{J}$.

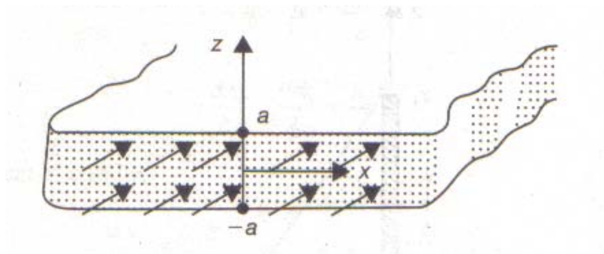

Figure 3: Region $-a \leq z \leq a$.

9. Given that the vector magnetic potential within a cylindrical conductor of radius a is $\vec{A} = -\frac{\mu_o I r^2}{4\pi a^2}\,\hat{a}_z$ find the corresponding $\vec{H}$.

10. One uniform current sheet, $\vec{K} = K_0\,(-\hat{a}_y)$, is located at $x = 0$ and another, $\vec{K} = K_0\,\hat{a}_y$, is at $x = a$. Find the vector potential between the sheets.

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FACULTAD DE INGENIERÍA, ARQUITECTURA Y DISEÑO

INGENIERÍA EN NANOTECNOLOGÍA

- CAMPOS ELECTROMAGNÉTICOS -

Problems 5: Magnetostatics 2

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1. A conductor 4 m long lies along the y axis with a current of 10.0 A in the $\hat{a}_y$ direction. Find the force on the conductor if the field in the region is $\vec{B} = 0.05\,\hat{a}_x$ T.

2. A conductor of length 2.5 m located at $z = 0$, $x = 4$ m carries a current of 12.0 A in the $-\hat{a}_y$ direction. Find the uniform $\vec{B}$ in the region if the force on the conductor is 1.20×10^{-2} N in the direction $(-\hat{a}_x + \hat{a}_z)\sqrt{2}$.

3. A current strip 2 cm wide carries a current of 15.0 A in the $\hat{a}_x$ direction, as show in Fig. 1. Find the force on the strip per unit length if the uniform field is $\vec{B} = 0.20\,\hat{a}_y$ T.

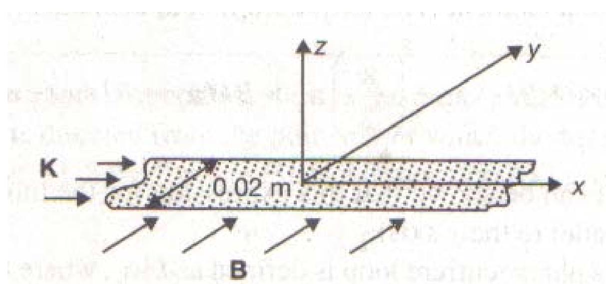

Figure 1: Current strip

4. Find the forces per unit length on two long, straight, parallel conductors if each carries a current of 10.0 A in the same direction and the separation distance is 0.20 m.

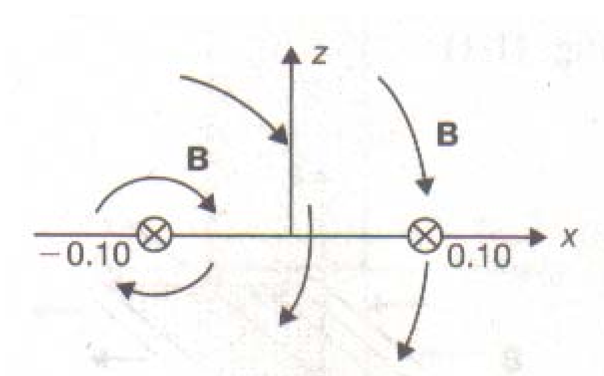

Figure 2: Parallel conductors

5. A conductor carries current I parallel to a current strip of density K_o and w, as shown in Fig. 3. Find an expression for the force per unit length on the conductor. What is the result when the width w approach infinity?

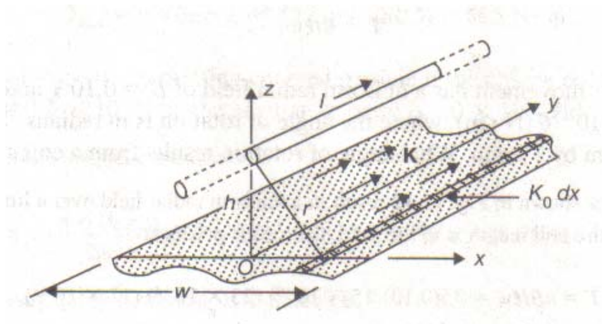

Figure 3: Conductor parallel to current strip

6. A conducting filament carries current I from point $A(0,0,a)$ to point $B(0,0,b)$. Show that at point $P(x,y,0)$,

$$\vec{H} = \frac{1}{4\pi\sqrt{x^2+y^3}}\left[\frac{b}{\sqrt{x^2+y^2+b^2}} - \frac{a}{\sqrt{x^2+y^2+a^2}}\right]\hat{a}_\phi$$

7. Find $\vec{H}$ at the center of a square current loop of side L.

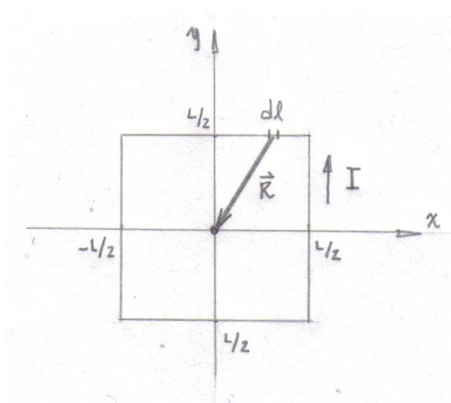

Figure 4: Square loop

8. Two identical circular current loops of radius $r = 3$ m and $I = 20$ A are in parallel planes, separated on their common axis by 10 m. Find $\vec{H}$ at a point midway between the two loops.

9. A radial field

$$\vec{H} = \frac{2.39 \times 10^6}{r\cos\phi}\,\hat{a}_r \text{ (A/m)}$$

exist in the free space. Find the magnetic flux Φ crossing the surface defined by $-\pi/4 \le \phi \le \pi/4$, $0 \le z \le 1$ m.

10. A current filament of 0.5 A in the $\hat{a}_y$ direction is parallel to the y axis at $x = 2$ m, $z = -2$ m. Find $\vec{H}$ at the origin.

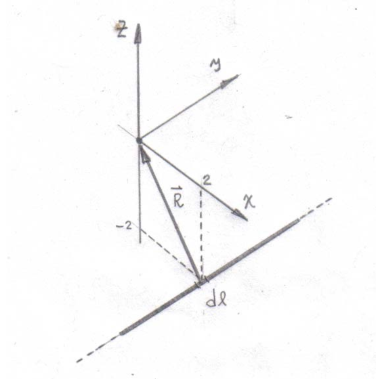

Figure 5: Current filament

11. In cylindrical coordinates, $\vec{B} = (2.0/r)\,\hat{a}_\phi$ (T). Determine the magnetic flux Φ crossing the plane surface defined by $0.5 \le r \le 2.5$ m and $0 \le z \le 2.0$ m.

12. Compute the total magnetic flux Φ crossing the $z = 0$ plane in cylindrical coordinates for $r \le 5 \times 10^{-2}$ m if
$$\vec{B} = \frac{2.0}{r}\sin^2\phi\,\hat{a}_z(\mathrm{T})$$

13. Given that
$$\vec{B} = 2.50(\sin\frac{\pi x}{2})e^{-2y}\,\hat{a}_z(\mathrm{T})$$
find the total magnetic flux crossing the strip $z = 0$, $y \ge 0$, $0 \le x \le 2$ m.

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FACULTAD DE INGENIERÍA, ARQUITECTURA Y DISEÑO
INGENIERÍA EN NANOTECNOLOGÍA
- CAMPOS ELECTROMAGNÉTICOS -

Problemas 6: Campo Magnético Estable.
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1. a) Encontrar $\vec{H}$ en componentes cartesianas en el punto $P(2,3,4)$ si hay una corriente filamentaria de 8 mA en el eje z en la dirección de $\hat{a}_z$. b) Repetir el problema si el filamento se encuentra en $x=-1$, $y=2$. c) Encontrar $\vec{H}$ si ambos filamentos están presentes.

2. Un conductor filamentario forma un triángulo equilátero cuyos lados son de longitud l y transporta una corriente I. Encontrar la intensidad del campo magnético en el centro del triángulo.

3. a) Un filamento forma un círculo de radio a, centrado en el origen sobre el plano $z=0$. Transporta una corriente I en la dirección de $\hat{a}_\phi$. Encontrar $\vec{H}$ en el origen. b) Un segundo filamento en forma de cuadrado está en el plano $z=0$. Los lados son paralelos a los ejes coordenados y una corriente I fluye en la dirección $\hat{a}_\phi$. Determine la longitud del lado b (en términos de a), tales que $\vec{H}$ en el origen tenga la misma magnitud que la del círculo del inciso a).

4. Los filamentos conductores paralelos mostrados en la Fig. 1 se encuentran en el espacio libre. Graficar $|\vec{H}|$ versus y, $-4 < y < 4$ a lo largo de la línea $x=0$, $z=2$.

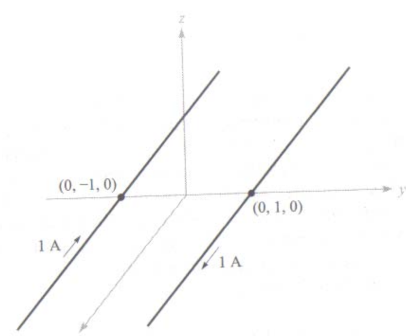

Figure 1: Filamentos conductores

5. Una placa de corriente $\vec{K} = 8\,\hat{a}_x$ A/m fluye en la región $-2 < y < 2$ en el plano $z=0$. Calcular $\vec{H}$ en $P(0,0,3)$.

6. Un cascarón conductor esférico hueco de radio a tiene conexiones filamentarias en la parte superior ($r=a, \theta=0$) y en la inferior ($r=a, \theta=\pi$). Una corriente directa I fluye desde el filamento superior, pasa por la superficie de la esférica y llega al filamento inferior. Encontrar $\vec{H}$ en coordenadas eféricas a) dentro y b) fuera de la esfera.

7. Un filamento infinito sobre el eje z transporta una corriente de 20π mA en la dirección $\hat{a}_z$. También están presentes tres placas de corrientes cilíndricas uniformes: 400 mA/m en $\rho = 1$ cm, -250 mA/m en $\rho = 2$ cm y -300 mA/m en $\rho = 3$ cm. Calcular H_ϕ en $\rho = 0.5$, 1.5, 2.5 y 3.5 cm.

8. Un cascarón cilíndrico hueco de radio a está centrado en el eje z y transporta una densidad de corriente superficial uniforme de $K_a\,\hat{a}_\phi$. *a*) Demostrar que $\vec{H}$ no es función de ϕ o de z. *b*) Demostrar que H_ϕ y H_ρ valen cero en cualquier lugar. *c*) Demostrar que $H_z = 0$ para $\rho > a$. *d*) Demostrar que $H_z = K_a$ para $\rho < a$. Una segunda placa, $\rho = b$, transporta una corriente de $K_b\,\hat{a}_\phi$. Encontrar $\vec{H}$ en cualquier punto.

9. Un toroide que tiene sección transversal de forma rectangular está definido por las siguientes superficies: los cilindros $\rho = 2$ y $\rho = 3$ cm, y los planos $z = 1$ y $z = 2.5$ cm. El toroide transporta una densidad de corriente de superficie de $-50\,\hat{a}_z$ A/m sobre la superficie $\rho = 3$ cm. Encontrar $\vec{H}$ en el punto $P(\rho, \phi, z)$: *a*) $P_A(1.5\text{ cm}, 0, 2\text{ cm})$; *b*) $P_B(2.1\text{ cm}, 0, 2\text{ cm})$; *c*) $P_c(2.7\text{ cm}, \pi/2, 2\text{ cm})$; *d*) $P_D(3.5\text{ cm}, \pi/2, 2\text{ cm})$.

10. Un cascarón cilíndrico definido por $1\text{cm} < \rho < 1.4\text{cm}$ consiste de un material conductor no magnético y transporta una corriente total de 50 A en la dirección $\hat{a}_z$. Encontrar el flujo magnético total que cruza el plano $\phi = 0$, $0 < z < 1$: *a*) $0 < \rho < 1.2$ cm; *b*) $1.0\text{ cm} < \rho < 1.4\text{ cm}$; *c*) $1.4\text{ cm} < \rho < 20\text{ cm}$.

UNIVERSIDAD AUTÓNOMA DE BAJA CALIFORNIA
FACULTAD DE INGENIERÍA, ARQUITECTURA Y DISEÑO
INGENIERÍA EN NANOTECNOLOGÍA
- CAMPOS ELECTROMAGNÉTICOS -

Problems 7: Time-Varying Fields
Dr. E. Efrén García G.

1. The bar conductor parallel to the y axis shown in Fig. 1 completes a loop by sliding contact with the conductors at $y = 0$ and $y = 0.05$ m. (a) Find the induced voltage when the bar is stationary at $x = 0.05$ m and $\vec{B} = 0.30 \sin 10^4 t\, \hat{a}_z$ (T). (b) Repeat for velocity of the bar $\vec{V} = 150\, \hat{a}_x$ (m/s). Discuss the polarity.

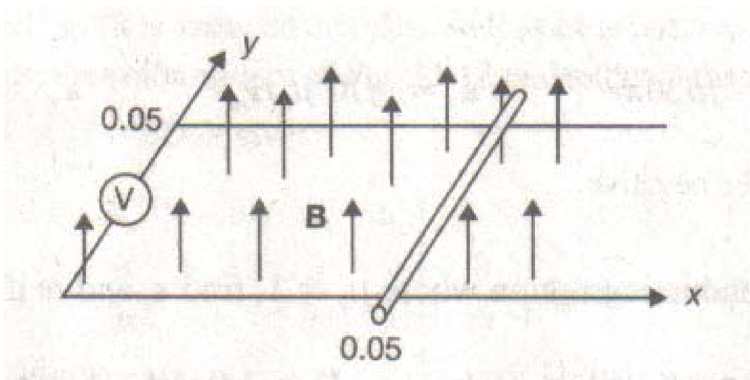

Figure 1: Bar conductor.

2. The rectangular coil in Fig. 2 moves to the right at speed $V = 2.5$ m/s. The left side cuts flux at right angles, where $B_1 = 0.30$ T, while the right side cuts equal flux in the opposite direction. Find the instantaneous current in the coil and discuss its direction by use de Lenz's law.

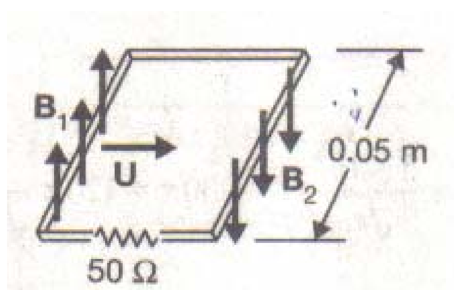

Figure 2: Rectangular coil.

3. A rectangular conducting loop in the $z = 0$ plane with sides parallel to the axes has y dimension 1 cm and x dimension 2 cm. Its resistance is 5.0 Ω. At a time when the coil side are at $x = 20$ cm and $x = 22$ cm it is moving toward the origin at a velocity of 2.5 m/s along the x axis. Find the current if $\vec{B} = 5.0e^{-10x}\, \hat{a}_z$ (T). Repeat for the coil sides at $x = 5$ cm and $x = 7$ cm.

4. The 2.0 m conductor shown in Fig. 3 rotates at 1200 rev/min in the radial field $\vec{B} = 0.10 \sin\phi\, \hat{a}_r$ (T). Find the current in the closed loop with a resistance of 100 Ω. Discuss the polarity and the current direction.

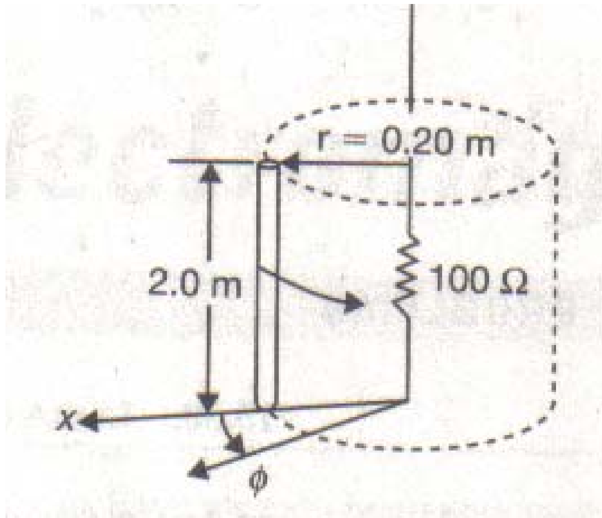

Figure 3: Conductor.

5. In a radial field $\vec{B} = 5.0\, \hat{a}_r$ (T), two conductors at $r = 0.23$ m and $r = 0.25$ m are parallel to the z axis and are 0.01 m in length. If both conductors are in the plane $\phi = 40\pi t$, what voltage is available to circulate a current when the two conductors are connected by radial conductors?

6. Given the conduction current density in a lossy dielectric as $J = 0.02 \sin 10^9 t$ (A/m^2), find the displacement current density if $\sigma = 10^3$ (S/m) and $\epsilon_r = 6.5$.

7. A circular-cross-section conductor of radius 1.5 mm carries a current $i_c = 5.5 \sin 4\times10^9 t$ (μA). What is the amplitude of the displacement current density, if $\sigma = 35$ (MS/m) and $\epsilon_r = 1$?

8. Concentric spherical conducting shell at $r_1 = 0.5$ mm and $r_2 = 1$ mm are separated by a dielectric for which $\epsilon_r = 8.5$. find the capacitance and calcutate i_c, given and applied voltage $v = 150 \sin 5000t$ (V). Obtain de displacement current i_D and compare it with i_c.

9. Two parallel conducting plates of area 0.05 m^2 are separate by 2 mm of a lossy dielectric for which $\epsilon_r = 8.3$ and $\sigma = 8.0 \times 10^{-4}$ (S/m).Given an applied voltage $v = 10 \sin 10^7 t$ (V), find the total rms current.

10. A conductor on the x axis between $x = 0$ and $x = 0.2$ m has a velocity $\vec{V} = 6.0\, \hat{a}_z$ (m/s) in a field $\vec{B} = 0.04\, \hat{a}_y$ (T). Find the induce voyage by using (a) the motional electric field intensity, (b) $d\phi/dt$, and (c) $B\ell V$. Determine the polarity and discuss Lenz's law if the conductor was connected to a closed loop.

UNIVERSIDAD AUTÓNOMA DE BAJA CALIFORNIA
FACULTAD DE INGENIERÍA, ARQUITECTURA Y DISEÑO
INGENIERÍA EN NANOTECNOLOGÍA
- CAMPOS ELECTROMAGNÉTICOS -

Problems 8: Electromagnetic Waves.
Dr. E. Efrén García G.

1. A carbon dioxide laser emits a sinusoidal electromagnetic wave that travels in vacuum in the negative $x-$direction. The wavelength is 10.6 μm (in the infrared, see FIG. 1 and the $\vec{E}$ field is parallel to the $z-$axis, with $E_{max} = 1.5$ MV/m. Write vector equations for $\vec{E}$ and $\vec{B}$ as a functions of time and position.

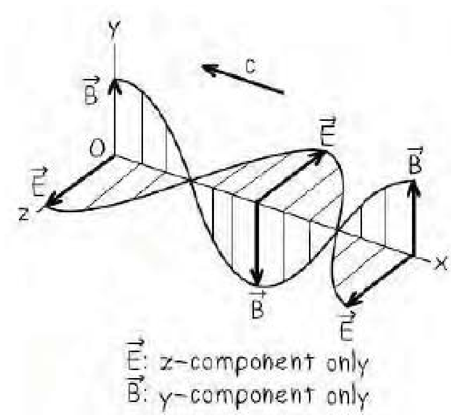

Figure 1: Sinusoidal electromagnetic wave.

2. (a) Visiting a jewelry store one evening, you hold a diamond up to light of sodium-vapor street lamp. The heated sodium vapor emits yellow light with frequency of 5.09×10^{14} Hz. Find the wavelength in vacuum and the wave speed and wavelength in diamond, for which $\epsilon_r = 5.84$ and $\mu_r = 1.00$ at this frequency.
   (b) A 90.0 MHz radio wave (in the FM radio band) passes from vacuum into an insulating ferrite (a ferromagnetic material used in computer cables to suppress radio interference). Find the wavelength in vacuum and the wave speed and wavelength in the ferrite, for $\epsilon_r = 10.0$ and $\mu_r = 1000$ at this frequency.

3. Suppose that $E = 100\text{V/m} = 100\text{N/C}$. Find the value of B, the energy density u, and the rate of energy flow per unit area S.

4. Calculate the intensity (the intensity I of the wave is the time-average value S_{av} of the magnitude of the Poynting vector $\vec{S}$) of the wave representation given by the equations:

$$E_y(x,t) = -2E_{max} \sin kx \sin \omega t$$
$$B_z(x,t) = -2B_{max} \cos kx \cos \omega t$$

5. Consider each of the electric- and magnetic-field orientations given next. In each case, what is the direction of propagation of the wave? (a) $\vec{E}$ in the $+x$-direction, $\vec{B}$ in the $+y$-direction; (b) $\vec{E}$ in the $-y$-direction, $\vec{B}$ in the $+x$-direction; (c) $\vec{E}$ in the $+z$-direction, $\vec{B}$ in the $-x$-direction; (d) $\vec{E}$ in the $+y$-direction, $\vec{B}$ in the $-z$-direction.

6. A sinusoidal electromagnetic wave is propagating in vacuum in the $+z$-direction. If at a particular instant and at a certain point in space the electric field is in the $+x$-direction and has magnitude 4.00 V/m, what are the magnitude and the direction of the magnetic field of the wave at this same point in space and instant in time?

7. The electric field of a sinusoidal electromagnetic wave obeys the equation: $E = (375(\mathrm{V/m})\cos[(1.99\times10^{7}\mathrm{rad/m})x + (5.97\times10^{15}\mathrm{rad/s})t]$. (a) What is the speed of the wave? (b) What are the amplitudes of the electric and magnetic fields of this wave? (c) What are the frequency, wavelength, and the period of the wave? Is this light visible to humans?

8. An electromagnetic wave with frequency 5.70×10^{14} Hz propagates with a speed of 2.17×10^{8} m/s in a certain piece of glass. Find (a) the wavelength of the wave in the glass; (b) the wavelength of a wave of the same frequency propagating in air; (c) the index of refraction n of the glass for an electromagnetic wave with this frequency; (d) the dielectric constant for glass at this frequency, assuming that the relative permeability is unity.

9. Scientists are working on a new technique to kill cancer cells by zapping them with ultrahigh-energy (in the range of 10^{12} W) pulses of light that last for an extremely short time (a few nanoseconds). These shorts pulses scramble the interior of a cell without causing it to explode, as long pulses would do. We can model a typical such cell as a disk 5.0 μm in diameter, with the pulses lasting for 4.0 ns with an average power of 2.0×10^{12} W. We shall assume that the energy is spread uniformly over the faces of 100 cells for each pulse. (a) How much energy is given to the cells during this pulses? (b) What is the intensity (in W/m^2) delivered to the cells? (c) What are the maximum values of the electric and magnetic fields in the pulse?

10. A sinusoidal electromagnetic wave from a radio station passes perpendicularly through an open window that has area 0.5 m^2. At the window, the electric field of the wave has rms value 0.0400 V/m. How much energy does this wave carry through the window durinf a 30.0 s commercial?

UNIVERSIDAD AUTÓNOMA DE BAJA CALIFORNIA
FACULTAD DE INGENIERÍA, ARQUITECTURA Y DISEÑO
INGENIERÍA EN NANOTECNOLOGÍA
- CAMPOS ELECTROMAGNÉTICOS -

Problemas 9: Electromagnetic Waves II.
Dr. E. Efrén García G.

1. Find the magnitudes of $\vec{D}$, $\vec{P}$, and ϵ_r, for a dielectric material in which $E = 0.15$ MV/m and $\chi_e = 4.25$.

2. Given $\vec{E} = -3\,\hat{a_x} + 4\,\hat{a_y} - 2\,\hat{a_z}$ V/m in the region $z < 0$, where $\epsilon_r = 2.0$, find $\vec{E}$ in the region $z > 0$, for which $\epsilon_r = 6.5$.

3. The plane interface between two dielectrics is given by $3x+z = 5$. On the side including the origin, $\vec{D} = (4.5\,\hat{a_x} + 3.2\,\hat{a_z})10^7$ and $\epsilon_{r1} = 4.3$, while on the other side $\epsilon_{r2} = 1.80$. Find E_1, E_2, D_2, and θ_2.

4. Find the capacitance of a parallel-plate capacitor with a dielectric of $\epsilon_r = 3.0$, area 0.92 m^2, and separation 4.5 mm.

5. Find the capacitance between the inner and outer curved conductor surfaces shown in FIG. 1. Neglect fringing.

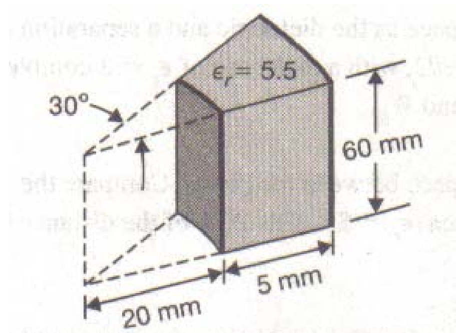

Figure 1: curved conductor surfaces.

6. A parallel-plate capacitor with area 0.30 m^2 and separation 5.5 mm contains three dielectrics with interfaces normal to $\vec{E}$ and $\vec{D}$, as follows: $\epsilon_{r1} = 3.0$, $d_1 = 1.0$ mm; $\epsilon_{r2} = 4.0$, $d_2 = 2.0$ mm; $\epsilon_{r3} = 6.0$, $d_3 = 2.5$ mm. Find the capacitance.

7. In free space, $\vec{D} = D_m \sin(\omega t + \beta z)\,\hat{a_x}$. Using Maxwell's equations, show that

$$\vec{B} = \frac{-\omega\mu_o D_m}{\beta}\sin(\omega t + \beta z)\,\hat{a_y}.$$

Sketch the fields at $t = 0$ along the z axis, assuming that $D_m > 0, \beta > 0$.

8. In homogeneous region where $\mu_r = 1$ and $\epsilon_r = 50$,

$$\vec{E} = 20\pi e^{i(\omega t - \beta z)}\, \hat{a_x} \quad (V/m), \qquad \vec{B} = \mu_o H_m e^{i(\omega t - \beta z)}\, \hat{a_y} \quad (T).$$

Find ω, and H_m if the wavelength is 1.78 m.

9. Given $\vec{E}(z,t) = 10^3 \sin(6 \times 10^8 t - \beta z)\, \hat{a_y}$ (V/m) in free space, sketch the wave at t=0 and at time t_1 when it has traveled $\lambda/4$ along the z-axis. Find t_1, β and λ.

10. Find the propagation constant at 400 MHz for a medium in which $\epsilon_r = 1.6$, $\mu_r = 4.5$, and $\sigma = 0.6$ S/m. Find the ratio of the velocity v to the free-space velocity c.

11. For silver, $\sigma = 3.0$ MS/m. At what frequency will the depth of penetration δ be 1 mm?

12. In free space, $\vec{H} = 0.1 \cos(2 \times 10^8 t - kx)\, \hat{a_y}$ A/m.

    (a) Calculate k, λ, and T.

    (b) Calculate the time t_1 it tkaes the wave to travel $\lambda/8$.

    (c) Sketch the wave at time t_1.

13. A plane wave propagating through a medium with $\epsilon_r = 8$, $\mu_r = 2$ has $\vec{E} = 0.5e^{-z/3} \sin(10^8 t - \beta z)\, \hat{a_x}$ V/m. Determine

    (a) β.

    (b) The loss tangent.

    (c) Intrinsic impedance.

    (d) Wave velocity.

    (e) $\vec{H}$ field.

14. A plane wave in nonmagnetic medium has $\vec{E} = 50 \sin(10^8 t + 2z)\, \hat{a_y}$ V/m. Find

    (a) The direction of wave propagation.

    (b) λ, f, and ϵ_r.

    (c) $\vec{H}$

15. A plane wave traveling in the $+y$-direction in a lossy medium ($\epsilon_r = 4, \mu_r = 1, \sigma = 10^{-2}$ S/m) has $\vec{E} = 30 \cos(10^9 \pi t + \pi/4)\, \hat{a_z}$ V/m at $y = 0$. Find

    (a) E at $y = 1$ m, $t = 2$ ns.

    (b) The distance traveled by the wave to have a phase shift of 10^o.

    (c) The distance traveled by the wave to have its amplitude reduced by 40%.

    (d) $\vec{H}$ at $y = 2$ m, $t = 2$ ns.